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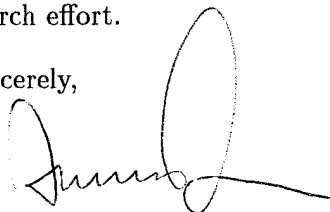
12 June 1995

Dr. Neal D. Glassman, Program Manager  
AFOSR/NM  
Air Force Office of Scientific Research  
110 Duncan Avenue, Suite 100  
Bolling AFB, DC 20332-0001

Dear Dr. Glassman:

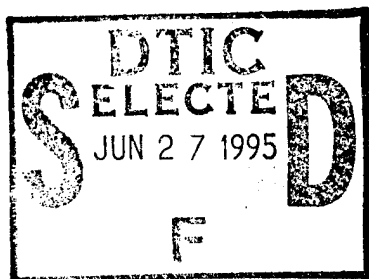
Attached in three copies is the Final Technical Report for AFOSR Grant F49620-93-1-0068.  
Thank you very much for your support of this research effort.

Sincerely,



Stephen M. Robinson  
Professor of Industrial Engineering  
and Computer Sciences

xc: Grants Administrative Office, AFOSR



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COMPUTATION AND THEORY IN NONLINEAR OPTIMIZATION

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13. ABSTRACT (Maximum 200 words)

The principal objectives of this research project were (1) developing path-following methods for normal-map equations, (2) exploring diffusion methods for global optimization and related stochastic methods, and (3) exploiting convex duality for applications in optimization such as decomposing optimization problems, as well as (4) use of the research results to improve problem-solving ability in applications areas. Progress was achieved in all of these areas, and was documented in nine papers prepared for publication in journals or in edited volumes, as well as one Ph.D. dissertation.

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## Final Technical Report

Computation and Theory in Nonlinear Optimization

Grant No. F49620-93-1-0068

University of Wisconsin-Madison

### 1. Abstract

The principal objectives of this research project were (1) developing path-following methods for normal-map equations, (2) exploring diffusion methods for global optimization, and related stochastic methods, and (3) exploiting convex duality for applications in optimization such as decomposing optimization problems, as well as (4) use of the research results to improve problem-solving ability in application areas. Progress was achieved in all of these areas, and was documented in nine papers prepared for publication in journals or in edited volumes, as well as one Ph.D. dissertation.

### 2. Overview of Research Accomplished

This overview is organized according to the major categories in the abstract above.

#### 1. *Developing path-following methods for normal-map equations.*

Activity in this area consisted of two branches, pursued simultaneously. The first was to extend the nonsmooth calculus previously developed for normal maps to provide existence and continuity results for the trajectories of solutions produced by a homotopy such as

$$H(x, t) = (1 - t)(x - a) + tf_C(x),$$

where  $f$  is a  $C^2$  function from  $\mathbf{R}^n$  to  $\mathbf{R}^n$ ,  $C$  is a polyhedral convex set in  $\mathbf{R}^n$ , and

$$f_C(x) = f(\Pi_C(x)) + (x - \Pi_C(x)).$$

Here  $\Pi_C(x)$  is the (Euclidean) projection of  $x$  on  $C$ .

This  $f_C$  is the *normal map* induced by  $f$  and  $C$ . If the homotopy level  $t = 1$  can be attained in the equation  $H(x, t) = 0$  then we will have found an  $x$  for which  $f_C(x) = 0$ , and then it can easily be shown that the point  $y = \Pi_C(x)$  satisfies

$$\text{For each } c \in C, \quad \langle c - y, f(y) \rangle \geq 0,$$

namely the *variational inequality problem* associated with  $f$  and  $C$ . These variational inequalities occur very frequently in modeling problems from operations research, engineering, and other areas. Therefore it is of great importance to have effective methods for solving them.

The existence and continuity results mentioned above appear in [D1], and papers based on that work are currently being prepared for publication. The other branch of work in

this area was computational implementation of nonsmooth homotopy methods. This work was also reported in [D1]. Programs based on it are now running and test problems are being analyzed. Computational results are expected to appear in future papers.

*2. Exploring diffusion methods for global optimization, and related stochastic methods.*

In this area some cooperative activity was carried out with the research group of Prof. Dr. K. Ritter at the Technical University of Munich, Germany. As a result of work done under the predecessor Air Force grant, several related stochastic areas were investigated to see if they could yield applicable results. Success was achieved in two distinct areas, one being the extension of methods for establishing large deviation bounds for probability distributions (reported in [P3]) and the other being the application of certain deterministic methods to stochastic optimization (reported in [P2], [P4], [P5], [P7], and [P9]). This work had unexpected, and very interesting, connections with discrete-event dynamic systems, and it has led to more effective methods for optimizing certain systems of that kind (for example, optimizing production lines with respect to machine cycle times) than were previously available.

*3. Exploiting convex duality for applications in optimization such as decomposing optimization problems.*

In this area revisions were carried out to the papers [P1] and [P6], which were based on work originally done under the predecessor grant. In addition, new methods using convexity techniques for sensitivity analysis of variational inequalities were explained in [P8]. Work continues on tying this area together with the normal-map methods discussed under (1) above.

*4. Improving problem-solving in applications areas.*

Of the papers listed below, [P1] shows how to compute solutions to large stochastic optimization problems often faced by planners. [P2], [P4], and [P5] give techniques that can be applied to optimize discrete-event dynamic systems occurring in practice, and [P6] gives a method for computational solution of normal-map equations; as indicated above, this provides a means of solving variational inequalities, which are important in many areas of practice.

### 3. Results from Research Activity

The following scientific works acknowledge support from Grant F49620-93-1-0068.

#### *a. Dissertation.*

- [D1] H. Sellami, *A Continuation Method for Normal Maps*. Ph.D. Dissertation, Departments of Mathematics and Industrial Engineering, University of Wisconsin-Madison, 1994.

#### *b. Papers.*

- [P1] B. J. Chun and S. M. Robinson, Scenario analysis via bundle decomposition, Accepted by *Annals of Operations Research*.
- [P2] G. Gürkan, A. Y. Özge, and S. M. Robinson, Sample-path optimization in simulation, In: J. D. Tew, M. S. Manivannan, D. A. Sadowski, and A. F. Seila, eds., *Proceedings of the 1994 Winter Simulation Conference*, pp. 247-254.
- [P3] P. E. Ney and S. M. Robinson, Polyhedral approximation of convex sets with an application to large deviation probability theory, Accepted by *Journal of Convex Analysis*.
- [P4] E. L. Plambeck, B.-R. Fu, S. M. Robinson, and R. Suri, Throughput optimization in tandem production lines via nonsmooth programming, In: J. M. Schoen, ed., *Proceedings of the 1993 Summer Computer Simulation Conference*, pp. 70-75. Society for Computer Simulation, San Diego, CA 1993.
- [P5] E. L. Plambeck, B.-R. Fu, S. M. Robinson, and R. Suri, Sample-path optimization of convex stochastic performance functions, Accepted by *Mathematical Programming*.
- [P6] S. M. Robinson, Newton's method for a class of nonsmooth functions, *Set-Valued Analysis* 2 (1994) 291-305.
- [P7] S. M. Robinson, Convergence of subdifferentials under strong stochastic convexity, Accepted by *Management Science*.
- [P8] S. M. Robinson, Sensitivity analysis of variational inequalities by normal-map techniques, Forthcoming in *Variational Inequalities and Network Equilibrium Problems*, eds. F. Giannessi and A. Maugeri, Plenum Publishing Corp., London, 1995.
- [P9] S. M. Robinson, Analysis of sample-path optimization. Submitted for publication and currently being revised for resubmission.

### 4. Participating Professionals

The following professional personnel received salary support from Grant F49620-93-1-0068.

- Bradbury Franklin, Research Assistant
- Gül Gürkan, Research Assistant
- A. Yonca Özge, Research Assistant
- Erica L. Plambeck, Undergraduate Assistant
- Stephen M. Robinson, Professor
- Hichem Sellami, Research Assistant

## **5. Degrees Awarded**

Hichem Sellami received the degree of Doctor of Philosophy (Mathematics and Industrial Engineering) in 1994. Erica L. Plambeck received the degree of Bachelor of Science, Industrial Engineering, in 1994. Among the other participants, Bradbury Franklin, Gül Gürkan, and A. Yonca Özge are continuing students in the doctoral program (in Industrial Engineering except for B. Franklin, who is in Mathematics), and are expected to receive degrees in the future.

## **6. Inventions and Patent Disclosures**

During the work under this grant, there were no inventions that appeared to have any patent possibilities. Other (non-patentable) discoveries are contained in the papers reported above.

## **7. Other Information**

Further information about any of the activities reported above, or other aspects of this research program, can be obtained from the principal investigator, Stephen M. Robinson, at the Department of Industrial Engineering, University of Wisconsin-Madison, 1513 University Avenue, Madison, WI 53706-1572, telephone (608) 263-6862, fax (608) 262-8454, email [smr@cs.wisc.edu](mailto:smr@cs.wisc.edu).